Randomized Rounding - Best Subset Selection Regression for Real Time Optimization

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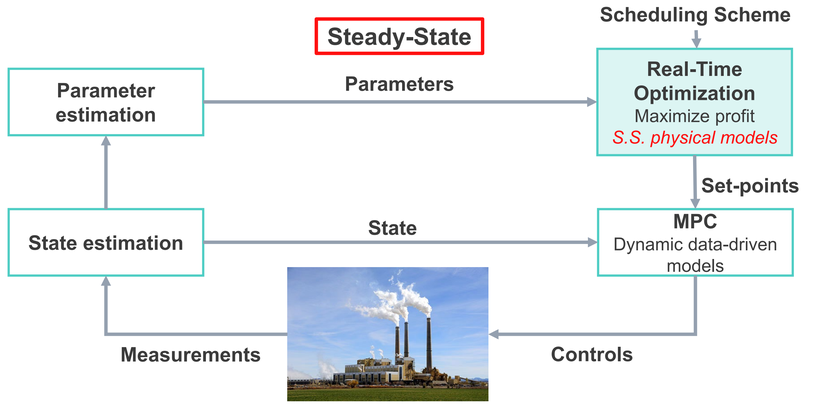
Models in Real Time Optimization



***Physical models** are complex and difficult to optimize-over, can we **build simple surrogates** to with requisite **accuracy** and **quickness** for RTO?*

Linear Model Building

$$\boldsymbol{y} = \sum_k \beta_k \boldsymbol{x_k} + \boldsymbol{\epsilon}$$

Response

$$\boldsymbol{y} \in \mathbb{R}^{n\times 1}$$

Regressors

$$\boldsymbol{x_k} \in \mathbb{R}^{n\times 1}$$
$$k \in K$$
$$|K| = p$$

Model-building goals:

- Simplicity
- Accuracy
- Speed

Best Subset Selection

$$\text{OBJ}(\beta, z) = \min_{\boldsymbol{\beta}, z} \sum_{i=1}^{n}(y_i - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda||\boldsymbol{\beta}||_0$$

- Seeks to find 'best subset' of regressors
- Computationally intensive (NP-hard)
- Recent progress in solving MIQP have allowed some problems to be solved exactly (e.g. **ALAMO**)
- Heuristic methods still dominant

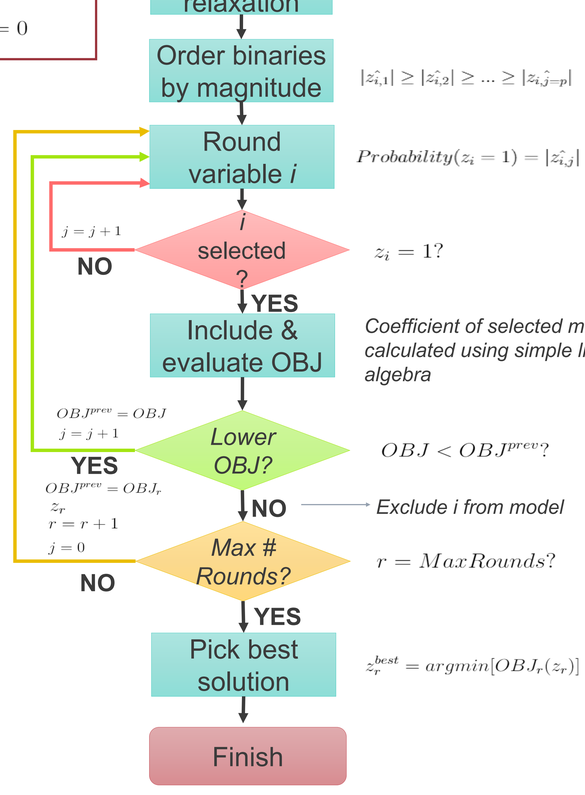
$$-Mz_j \le \beta_j \le Mz_j$$
$$z_j \in \{0,1\}^p \quad \forall j \in P$$

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 \rightarrow y = \beta_1 x_1 + \beta_3 x_3 + \beta_6 x_6$$

ALAMO
a black-box modeling tool

Randomized Rounding (RR FSS)

Current Minimum Objective	$OBJ^{prev} = \infty$
Variable Magnitude Rank	$j = 0$
Number of Iterations	$r = 0$

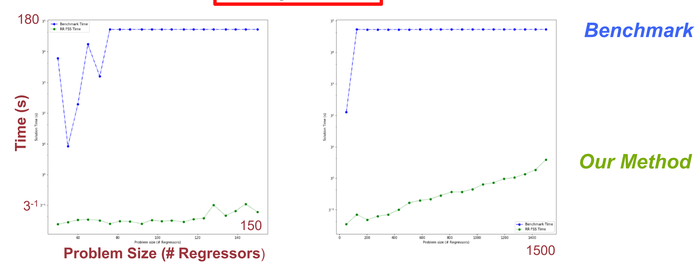


Experimental Questions

How does RR FSS compare with solving the MIQP 'exactly'?

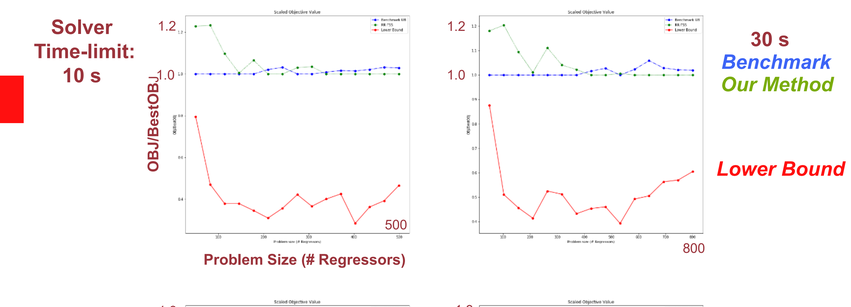
Speed

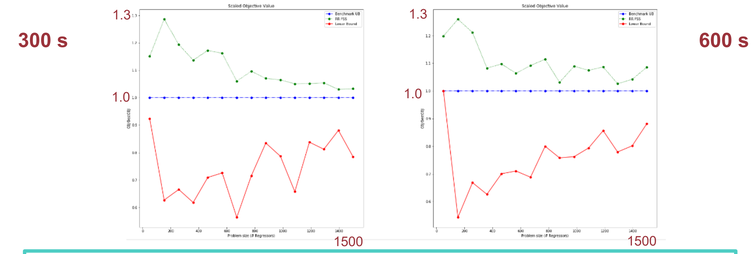
- Much faster



Objective Values

- Competitive





Do MIQP formulation hyper-parameters affect the results?

(Heuristic OBJ)/(Benchmark OBJ): $p = 500, s = 10, SNR = 0.25$

		$\hat{\lambda}$		
		0.05	0.1	0.15
$\hat{M}$	3	1.17	1.11	1.05
	3.5	1.25	1.10	1.04
	4	1.2	1.01	1.05

(Heuristic OBJ)/(Benchmark OBJ): $p = 1500, s = 10, SNR = 0.25$

		$\hat{\lambda}$		
		0.05	0.1	0.15
$\hat{M}$	3	1.00	0.83	0.88
	3.5	0.99	0.81	0.90
	4	1.19	0.91	0.91

Conclusions and Beyond

Conclusions:

- Randomized rounding for best subset selection is a polynomial-time algorithm for constructing good-quality regression models quickly and robustly

Upcoming work:

- Test our approach in a simulated RTO-framework

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